The Central Mass and Phase-Space Densities of Dark Matter Halos: Cosmological Implications

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ABSTRACT

Current data suggest that the central mass densities ρ_0 and phase-space densities $Q \equiv \rho_0/\sigma_V^3$ of cosmological halos in the present universe are correlated with their velocity dispersions σ_V over a very wide range of σ_V from less than 10 to more than 1000 $\mathrm{km\,s^{-1}}$. Such correlations are an expected consequence of the statistical correlation of the formation epochs of virialized objects in the CDM model with their masses; the smaller-mass halos typically form first and merge to form larger-mass halos later. We have derived the $Q-\sigma_V$ and $\rho_0-\sigma_V$ correlations for different CDM cosmologies and compared the predicted correlations with the observed properties of a sample of low-redshift halos ranging in size from dwarf spheroidal galaxies to galaxy clusters. Our predictions are generally consistent with the data, with preference for the currently-favored, flat ΛCDM model. Such a comparison serves to test the basic CDM paradigm while constraining the background cosmology and the power-spectrum of primordial density fluctuations, including larger wavenumbers than have previously been constrained.

Subject headings: cosmology: theory – dark matter – galaxies: formation – galaxies: halos – galaxies: clusters – galaxies: kinematics and dynamics

1. Introduction

Galaxies and clusters emerge in the CDM model when primordial density fluctuations grow to form dark-matter dominated halos in approximate virial equilibrium. For Gaussian-random-noise density fluctuations in a cosmologically-expanding, pressure-free gas of collisionless matter like standard CDM, gravitational instability assembles these halos hierarchically. Smaller-mass objects form first, then merge to form larger-mass objects, in a continuous sequence of increasing mass. As a result, a statistical correlation between the mass and collapse time of the objects which form is expected which depends upon the shape and amplitude of the primordial power spectrum $P(k) = |\delta_k|^2$ and on the rates of fluctuation growth in different background cosmologies. This suggests that a measurement of this correlation can constrain $P(k)$ and the fundamental cosmological parameters, while testing the CDM model.

The power spectrum is already well constrained at large scales by observations of the anisotropy of the Cosmic Microwave Background (CMB) and the mass function of galaxy clusters (Bahcall et al. 1999). At large wavenumber k, however, $P(k)$ is still poorly constrained. Recent CMB anisotropy measurements suggest that the universe is flat (e.g. de Bernardis et al. 2000, Hanany et al. 2000, Pryke et al. 2001). The measured distance-redshift relation for type Ia SNe (Perlmutter et al. 1999, Riess et al. 2000; Reiss et al. 2001; Turner and Riess 2001) combines with this to favor a low-density universe with a cosmological constant or some other "dark energy". It is important to establish whether the statistical correlation of halo mass and formation epoch predicted at large k by the standard CDM model is consistent with these constraints on $P(k)$ at small k and on the fundamental cosmological parameters.

Despite its many successes, the standard CDM model presently faces significant problems (e.g. Moore 2001 and refs. therein). For example, CDM simulations find singular halo density profiles and high abundance of subhalos within halos, in apparent conflict with observations, and have difficulty explaining galactic rotation. Recently, modifications of the standard CDM model have been proposed to resolve some of these problems, including the suggestion of self-interacting dark matter (SIDM) (Spergel & Steinhardt 2000) and other CDM variants (e.g. see Wandelt et al. 2000 and refs. therein). In addition to testing the standard CDM paradigm while further constraining $P(k)$ at large wavenumber, therefore, a measurement of the statistical correlations among the internal properties of halos over a wide mass range from dwarf spheroidal (dSph) galaxies ($M_{\rm halo} \sim 10^6 - 10^7 M_\odot$) to galaxy clusters ($M_{\rm halo} \sim 10^{14} - 10^{15} M_\odot$), expected if halo masses correlate with their formation epochs, would sharpen this debate.

For example, it was recently pointed out (Sellwood 2000, Dalcanton & Hogan 2000,

Hogan and Dalcanton 2000, Madsen 2001) that the correlation of the 1D velocity dispersion σ_V of a halo with the maximum value of its phase space density $f_{\rm max} \sim Q \equiv \rho_0/\sigma_V^3$, where ρ_0 is the central density of the halo, is a potent diagnostic of the properties of the dark matter. As argued in Sellwood (2000), Liouville's theorem applied to collisionless dark matter guarantees that processes like baryonic dissipation will not change $f_{\rm max}$ significantly. In that case, $f_{\rm max}$ can be used as a diagnostic even for halos that are no longer dark-matter dominated in the central region following baryon cooling and compression. Of related interest, there are recent claims that the core densities of all galaxy and cluster halos are approximately the same, independent of the mass of the halo (Firmani et al. 2000, Avila-Reese et al. 2000, Kaplinghat, Knox, & Turner 2000). A central density which is the same for halos of different mass would impose strong constraints on the nature of the dark matter, if this hypothesis is correct.

In what follows, we predict correlations of the maximum phase-space densities Q and of the central densities ρ_0 of cosmological halos with their velocity dispersions σ_V, for different CDM models. We will show that these predicted correlations are consistent with the observed properties of halos spanning a very large range of mass scales from dSph galaxies to galaxy clusters, and that the current data has the power to constrain the background universe and power spectrum of the CDM model. Finally, we shall discuss the hypothesis that halos have a core density that is independent of their mass.

2. The $Q - \sigma_V$ and $\rho_0 - \sigma_V$ Correlations for Cosmological Halos Observed Today

We have developed an analytical model for the postcollapse equilibrium structure of virialized objects which condense out of a cosmological background universe, either matter-dominated or flat with a cosmological constant (Shapiro, Iliev & Raga 1999, "Paper I"; Iliev & Shapiro 2001b, "Paper II"). The model is based upon the assumption that cosmological halos form from the collapse and virialization of "top-hat" density perturbations and are spherical, isotropic, and isothermal. This leads to a unique, nonsingular, truncated isothermal sphere (TIS), a particular solution of the Lane-Emden equation (suitably modified when $\Lambda \neq 0$). The size r_t and velocity dispersion σ_V are unique functions of the mass and redshift of formation of the object for a given background universe. Our TIS density profile flattens to a constant central value, ρ_0, which is roughly proportional to the critical density of the universe at the epoch of collapse, with a small core radius $r_0 \approx r_t/30$ (where $\sigma_V^2 = 4\pi G\rho_0 r_0^2$ and $r_0 \equiv r_{\rm King}/3$, for the "King radius" $r_{\rm King}$, defined by Binney and Tremaine, 1987, p. 228).

This TIS model reproduces many of the average properties of the halos in CDM N-

body simulations quite well, suggesting that it is a useful approximation for the halos which result from more realistic initial conditions. We have elsewhere compared this model with simulation results and applied it successfully in a variety of ways, including the derivation of the mass-radius-temperature virial relations and integrated mass profiles of X-ray clusters deduced from numerical CDM simulations by Evrard, Metzler & Navarro (1996) and Mathiesen & Evrard (2001) (Papers I, II), the rotation curves of dark-matter dominated dwarf galaxies and the observed correlation between their maximum velocity, $v_{\rm max}$, and the radius, $r_{\rm max}$, at which it occurs (Iliev & Shapiro 2001a), and the mass model of cluster CL 0024 determined by strong gravitational lensing measurements (Shapiro & Iliev 2000). The TIS mass profile agrees well with the fit to N-body simulations by Navarro, Frenk & White (1996; "NFW") (i.e. fractional deviation of $\sim 20\%$ or less) at all radii outside of a few TIS core radii (i.e. outside a King radius or so), for NFW concentration parameters $4 \leq c_{\rm NFW} \leq 7$. As a result, the TIS central density ρ_0 can be used to characterize the core density of cosmological halos, even if the latter have singular profiles like that of NFW, as long as we interpret ρ_0, in that case, as an average over the innermost region.

The TIS model yields the central density ρ_0, velocity dispersion σ_V, size r_t and core radius r_0 for any given halo as unique functions of its mass M and collapse redshift $z_{\rm coll}$. This defines a "cosmic virial plane" in the three-dimensional space of halo parameters (ρ_0, r_0, σ_V) [i.e. $2\ln\sigma_V - \ln\rho_0 - 2\ln r_0 - \ln(4\pi G) = 0$], in the terminology of Burstein et al. (1997), and determines the halo size, mass and collapse redshift for each point on this infinite plane, the ensemble of all possible virialized objects. In hierarchical models of structure formation like CDM, however, the statistical correlation of halo mass and collapse redshift determines the expected distribution of points on this cosmic virial plane. We shall combine our TIS model with the well-known Press-Schechter (PS) approximation (Press & Schechter 1974) for $z_{\rm coll}(M)$ – the typical collapse epoch for a halo of mass M – to derive the statistical distribution for the $Q - \sigma_V$ and $\rho_0 - \sigma_V$ correlations, as follows.

According to PS, the fraction of matter in the universe which is condensed into objects of mass $\geq M$ at a given epoch is erfc $\left(\nu/2^{1/2}\right)$, where $\nu \equiv \delta_{\rm crit}/\sigma(M)$, $\sigma(M)$ is the standard deviation of the density fluctuations at that epoch, according to linear theory, when filtered on mass scale M, and $\delta_{\rm crit}$ is the fractional overdensity of a top-hat perturbation when this linear theory is extrapolated to the time of infinite collapse in the exact nonlinear solution. The "typical" collapse epoch for a given mass is that for which $\sigma(M) = \delta_{\rm crit}$ (i.e. $\nu = 1$). For a given $z_{\rm coll}$, this defines a typical mass scale: $M_\star \equiv M(\nu = 1)$.

For any ν, the PS prescription above yields a unique $z_{\rm coll}$ for halos of a given mass, thus completely determining the $Q - \sigma_V$ and $\rho_0 - \sigma_V$ correlations. The TIS+PS predictions for the $Q - \sigma_V$ correlation for halos observed today are shown in Figure 1, for $\nu = 0.5 - 3$. For any

ν, there is some value of $\sigma_V = \sigma_{V,0}$ for which $z_{\rm coll} = 0$. For fluctuations which become halos observed today with $\sigma_V \geq \sigma_{V,0}$, the most likely collapse epoch is assumed to be $z_{\rm coll} = 0$. Hence, curves of constant ν merge with the straight line for $z_{\rm coll} = 0$ for this range of σ_V values.

For comparison, the observed properties of a sample of low-redshift, dark-matter dominated halos of galaxies and clusters are also plotted in Figure 1, using data from the following sources. Kormendy and Freeman (1996; 2001) have compiled and interpreted mass models fitted to galaxy rotation curves for 49 late-type spirals of type Sc-Im and velocity dispersions and core radii for 7 dSph galaxies, chosen to minimize the need to correct the deduced halo parameters for the effects of baryonic dissipation. Data for one additional dSph galaxy, Leo I, is from Mateo et al. (1998). For low-redshift clusters, we have adopted central total mass densities which are based upon the central gas densities and baryonic-gas-mass fractions derived from X-ray brightness profile fits for 28 nearby clusters by Mohr, Mathiesen & Evrard (1999), for which velocity dispersions were tabulated by Girardi et al. (1998) (26 clusters) and Jones & Forman (1999) (2 clusters). The galaxy and cluster halo profiles adopted by these authors in fitting data to derive their central densities are very similar to the TIS halo profile we have used to make our predictions, so these central densities are suitable for direct comparison.

The agreement in Figure 1 between our TIS+PS model predictions and the observed halos is excellent for the entire range of halo masses from dSph galaxies to galaxy clusters. The observed scatter of the data is natural from a theoretical point of view, due to the Gaussian random statistics of the initial density fluctuations. A comparison of results for different models suggests that the currently-favored, flat ΛCDM model ($\Omega_0 = 0.3, \lambda_0 = 0.7, h = 0.65$) is preferred. The COBE-normalized, low-density, matter-dominated OCDM model ($\Omega_0 = 0.3, \lambda_0 = 0, h = 0.65$), with a primordial power spectrum index of $n_{\rm p} = 1.3$, a tilt which allows it to match the galaxy cluster abundance at $z = 0$ (e.g. Wang et al. 2000), requires that most galaxy halos observed today be low-ν (i.e. late-collapsing), rather than typical. This may indicate that $P(k)$ for this model is too large on galactic mass scales. The untilted OCDM model ($n_{\rm p} = 1$), which predicts an incorrect cluster abundance at $z = 0$ (i.e. not "cluster-normalized"), requires, instead, that all halos observed today be high-ν, corresponding to rare, high-density peaks of the primordial density field. In that case, $P(k)$ may be too small on cluster mass scales and below. Finally, the cluster-normalized Einstein-de Sitter (EdS) model ("SCDM") shows good agreement with this halo data on galaxy scales, but there are many cluster data points for which the Q-values fall below the limiting values predicted to correspond to $z_{\rm coll} = 0$.

In Figure 2, we compare the TIS+PS prediction for the $\rho_0 - \sigma_V$ correlation with the same

halo data. Once again, the data are generally consistent with the theoretical prediction, with the same trends for different CDM models as identified above for the $Q - \sigma_V$ correlation. Hence, the flat, ΛCDM model is preferred. The data are clearly inconsistent, however, with the constant-core-density hypothesis, since observed core-densities show a clear downward trend from dSph galaxies to larger galaxies and clusters, decreasing by more than three orders of magnitude as σ_V increases by two orders of magnitude. The appearance of a core-density that is approximately independent of σ_V at large values of σ_V (i.e. from massive galaxy to cluster scales) is easily understood in the TIS+PS model if the Gaussian statistics of fluctuation amplitudes are considered. Since large galaxies and galaxy clusters observed today formed from high-ν fluctuations, the most probable time for them to collapse is close to the present. Therefore, our model predicts them all to have similar central densities, since ρ_0 is proportional to the mean density of the universe at the collapse epoch.

This success of the TIS+PS model in predicting the observed correlations can be understood by an analytical argument, as follows. According to Paper II, the exact dependences of the TIS model core densities and velocity dispersions on M and $z_{\rm coll}$ are well approximated by $\rho_0 = 1.80 \times 10^4 [F(z_{\rm coll})]^3 \rho_{\rm crit,0}$ and $\sigma_V^2 = 1.1 \times 10^4 (M/10^{12} M_\odot h^{-1})^{2/3} F(z_{\rm coll})\, {\rm km^2\, s^{-2}}$, where $\rho_{\rm crit,0} \equiv 3H_0^2/(8\pi G)$ and the function $F(z_{\rm coll})$ is defined by equation (84) of Paper II. The quantity F^3 is just the density of a top-hat perturbation after it collapses and virializes in the standard uniform sphere (SUS) approximation (as measured in units of the EdS value for $z_{\rm coll} = 0$, $18\pi^2 \rho_{\rm crit,0}$). For the EdS case, $F = (1 + z_{\rm coll})$, while for the low-density, matter-dominated and flat cases, $F \to \Omega_0^{1/3}(1 + z_{\rm coll})$ for early collapse. If we approximate the power-spectrum of density fluctuations at high redshift (e.g. just after recombination) as a power-law in wavenumber k, $P(k) \propto k^n$, and define a mass $M \propto k^{-3}$, then $F \propto M^{-(3+n)/6}$ if $n = n_{\rm eff} \equiv -3(2y_F + 1)$, where $y_F \equiv (d\ln F/d\ln M)_{\rm exact}$ at the relevant mass scale. For all masses in the EdS case and for masses which collapse early in the low-density, matter-dominated and flat, universes, y_F reduces to $y_\sigma \equiv (d\ln\sigma/d\ln M)_{\rm exact}$, and $(1 + z_{\rm coll}) \propto \sigma(M) \propto M^{-(3+n)/6}$, where $\sigma(M)$ is evaluated at the same cosmic time for all masses. The dependence of $n_{\rm eff}$ and $z_{\rm coll}$ on M for 1-σ fluctuations is shown in Figure 3 for ΛCDM, along with the approximate $n_{\rm eff}$ which results if y_F is replaced by y_σ, which shows that the latter is a very good approximation for all masses $M < 10^{12} M_\odot h^{-1}$. In terms of this power-law model, the TIS model then yields $\rho_0 \propto M^{-(n+3)/2}$, $\sigma_V \propto M^{(1-n)/12}$, and $Q \propto M^{-(n+7)/4}$, which combine to give

$$Q = Q_\star (\sigma_V/\sigma_{V,\star})^{\alpha-3}, \tag{1}$$

and

$$\rho_0 = \rho_{0,\star}(\sigma_V/\sigma_{V,\star})^{\alpha}, \tag{2}$$

where $\alpha \equiv 6(n+3)/(n-1)$, except that $\alpha = 0$ for $M > M_\star(z = 0)$, for which $z_{\rm coll} = 0$ is

assumed.

For the currently-favored, flat, ΛCDM model, for example, $n_{\rm eff} \approx -2.6, -2.4, -2.2$ for $M = M_\star = 10^7, 10^{10}, 10^{12} M_\odot h^{-1}$, respectively, and, hence, $\alpha \approx -0.7, -1.1, -1.8$ for these masses, while $\sigma_{V,\star} = 4.5, 33, 114\,{\rm km\,s^{-1}}$, $Q_\star = 1.4{\times}10^{-3}, 5.7{\times}10^{-7}, 2.4{\times}10^{-9}\,M_\odot{\rm pc}^{-3}({\rm km\,s^{-1}})^{-3}$, and $\rho_{0,\star} = 0.12, 0.02, 0.0035\,M_\odot{\rm pc}^{-3}$, respectively. For all masses $M > M_*(z=0) \approx 10^{13} h^{-1} M_\odot$, our model predicts $\rho_0(z_{\rm coll}=0) = 0.00143\,M_\odot{\rm pc}^{-3}$, and $Q(z_{\rm coll}=0) = \rho_0(z_{\rm coll}=0)/\sigma_V^3$.

Dalcanton and Hogan (2000) predict that $Q \propto \sigma_V^{-3}$ for all σ_V, which in our model is achieved only for $M \to 0$, for which $n_{\rm eff} = -3$, and in the high-mass limit, for which $z_{\rm coll} \approx 0$ typically. Similarly, according to equation (2), the TIS + PS model predicts that the core-density is the same for halos with different σ_V only in the small-mass limit in which $n_{\rm eff} = -3$ and the high-mass limit, for which $z_{\rm coll} \approx 0$, with a large difference between the values of ρ_0 in these two limits. The generally good agreement between this prediction and the data, as plotted in Figure 2, argues against the hypothesis that all halos share a single, universal core density. The putative universal core density may have escaped detection by the observations quoted here if it is confined to a region interior to the smallest radii probed by those observations, but it must be at least as large as the highest central densities reported in Figure 2. These high central densities are well in excess, of the universal core density of $0.02 M_\odot{\rm pc}^{-3}$ claimed by Firmani et al. (2000).

In the future, as more and better data become available, it should be possible to refine the comparison between the correlations predicted here and the observed properties of galaxies and clusters. We have demonstrated that such a correlation is expected, that current data are consistent with the predicted correlations for the CDM model, with preference for the currently-favored ΛCDM model over CDM in other background cosmologies, and that no significant deviation of the primordial power-spectrum shape from the scale-invariant, Harrison-Zeldovich shape ($n_p = 1$), the standard prediction of inflationary cosmology, is required. In addition, we have demonstrated that the data do not support the idea that all halos have the same core density, independent of their mass or formation epoch.

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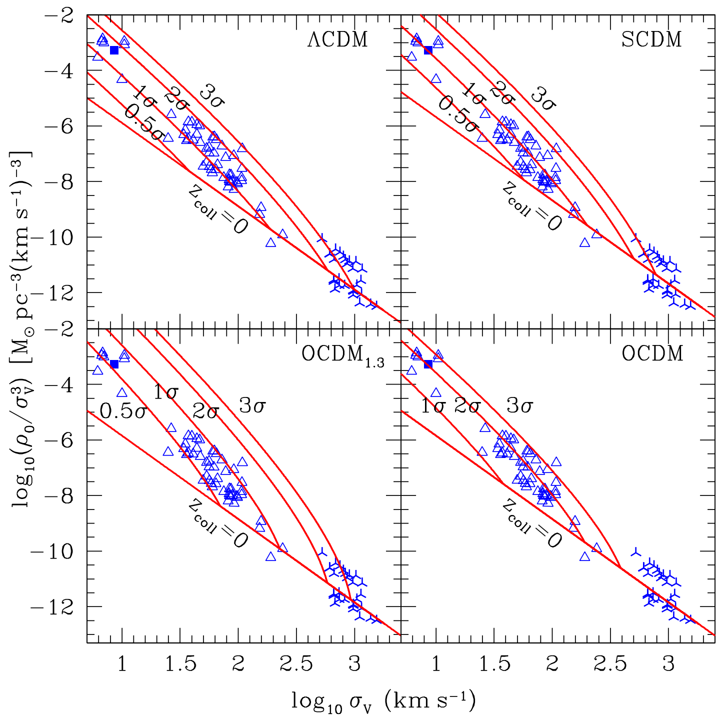
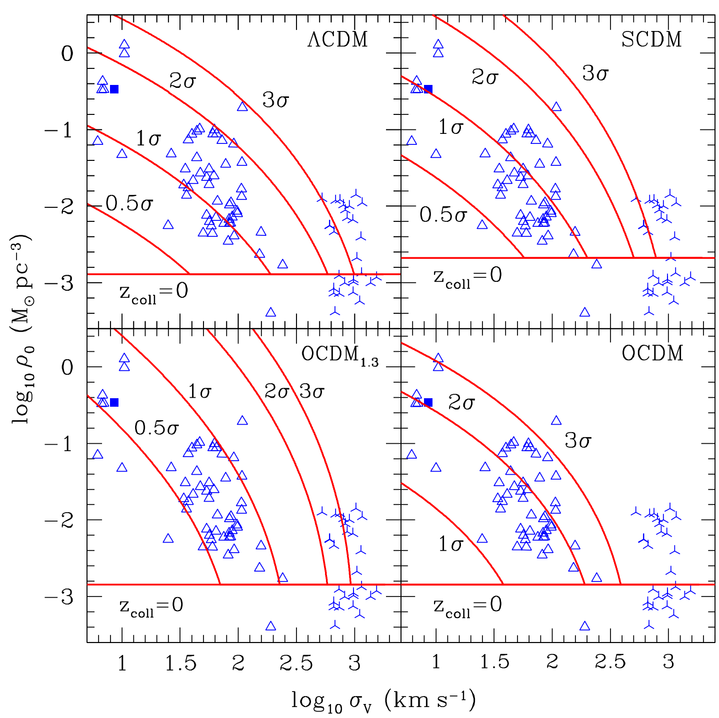
Fig. 1.— Maximum phase-space density $Q = \rho_0/\sigma_V^3$ versus velocity dispersion σ_V for halos observed today, as predicted for various CDM universes by the TIS + PS model (solid curves) for halos formed from ν-σ fluctuations, as labelled with the values of ν, for ν = 0.5, 1, 2, and 3. In each panel, lines representing halos of different mass which collapse at the same redshift are shown for the case $z_{\rm coll} = 0$, as labelled. Each panel represents different assumptions for the background universe and primordial density fluctuations, as labelled: COBE-normalized ΛCDM ($\lambda_0 = 0.7$, $\Omega_0 = 0.3$) (upper left), cluster-normalized

SCDM ($\Omega_0 = 1$) (upper right), and COBE-normalized OCDM ($\Omega_0 = 0.3$) (lower panels), all assuming $h = 0.65$ and primordial power spectrum index n_p =1 (i.e. untilted), except for OCDM$_{1.3}$, for which n_p =1.3. Data points represent observed galaxies and clusters, taken from the following sources: (1) galaxies from Kormendy & Freeman (1996; 2001) (open triangles); (2) Local Group dwarf galaxy Leo I from Mateo et al. (1998) (filled square); (3) cluster velocity dispersions from Girardi et al. (1998) and Jones & Forman (1999), central densities from Mohr et al. (1999) (crosses).

Fig. 2.— Same as Figure 1, but for central density ρ_0 versus velocity dispersion σ_V.

Fig. 3.— (a)(upper) Effective logarithmic slope, $n_{\rm eff}$, of the density fluctuation power spectrum versus halo mass $M = M_\star$ (i.e. for 1-σ fluctuations) for ΛCDM, based upon y_F (solid curve) and the approximation which uses y_σ, instead (dashed). (b) (lower) Typical collapse redshift $z_{\rm coll}$ for halos of mass $M_\star$ in ΛCDM.


ΛCDM
SCDM
OCDM$_{1.3}$
OCDM
0.5σ
1σ
2σ
3σ
$z_{coll}=0$
$\log_{10}(\rho_0/\sigma_V^3)$ [$M_\odot$ pc^{-3}(km s^{-1})$^{-3}$]
$\log_{10}\sigma_V$ (km s^{-1})


ΛCDM
SCDM
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0.5σ
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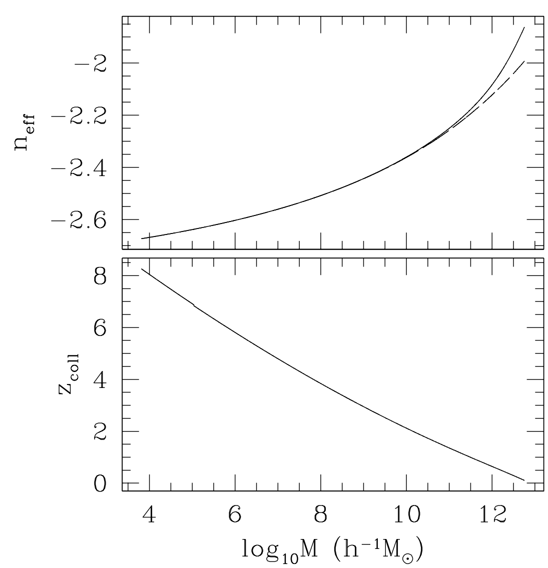

n_{eff}
−2
−2.2
−2.4
−2.6
z_{coll}
8
6
4
2
0
4
6
8
10
12
$\log_{10}M$ ($h^{-1}M_\odot$)